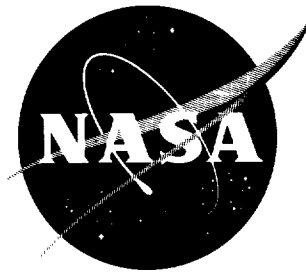


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# TECHNICAL NOTE

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## A REVIEW OF GEODETIC PARAMETERS

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# A REVIEW OF GEODETIC PARAMETERS

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## SUMMARY

It is recommended that the parametric values which are currently most used in orbital computation be adopted as provisional standards, rather than those which may be the best available, because the "most used" values differ only slightly from the "best" values and further improvements in the values are expected within the next 4 years. Some of these values are:

$$GM_{\oplus} = 3.986032 \times 10^{20} \text{ cm}^3 \text{ sec}^{-2},$$

$$J_2 = 1082.30 \times 10^{-6},$$

$$J_3 = -2.3 \times 10^{-6},$$

$$J_4 = -1.8 \times 10^{-6},$$

$$a_e = 6,378,165.0 \text{ m.}$$

With parameters such as the foregoing the most serious geodetic errors affecting astronomy are tracking station positions. Standard methods of describing and transforming positions are suggested.



# A REVIEW OF GEODETIC PARAMETERS\*

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## INTRODUCTION

This review recommends which geodetic parameters should be adopted as standard, the manner in which the parameters should be expressed, and the values which should be adopted. In making these recommendations, current practice, available determinations, and anticipated improvements will be considered.

## GRAVITATIONAL PARAMETERS

For the notation of the earth potential, recommendations have already been made by Commission 7 on Celestial Mechanics, of the International Astronomical Union (Reference 1):

$$U = \frac{\mu}{r} \left[ 1 + \sum_{n=1}^{\infty} \sum_{m=0}^n \left( \frac{R}{r} \right)^n P_n^m(\sin \beta) (C_{n,m} \cos m\lambda + S_{n,m} \sin m\lambda) \right], \quad (1)$$

where  $\mu = GM_{\oplus}$ ,  $r$  is the distance from the center of the earth,  $R$  is the mean equatorial radius of the earth,  $P_n^m$  is the associated Legendre polynomial,  $\beta$  is the latitude, and  $\lambda$  is the longitude. Alternative notations recommended for the gravitational coefficients are

$$J_n = -C_{n,0}, \quad (2)$$

and

$$(A_{n,m}, B_{n,m}) = \left[ \frac{(n+m)!}{(n-m)!} \right]^{1/2} (C_{n,m}, S_{n,m}). \quad (3)$$

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These two additions are suggested:

1. Define

$$(\bar{C}_{n,m}, \bar{S}_{n,m}) = \left[ \frac{(n+m)!}{(n-m)! (2n+1) (2-\delta_m^0)} \right]^{1/2} (C_{n,m}, S_{n,m}), \quad (4)$$

where the Dirac delta  $\delta_m^0$  is 1 for  $m = 0$  and 0 for  $m \neq 0$ . The  $\bar{C}_{n,m}$ ,  $\bar{S}_{n,m}$  are coefficients of harmonics which have a mean square amplitude of 1 for all values of  $n$  and  $m$ .

2. Define the mean equatorial radius more precisely as the equatorial radius of the mean earth ellipsoid, i.e., the ellipsoid of revolution which best fits the geoid. This definition is consistent with geodetic practice and involves the equatorial radius with only two of the set of orthogonal parameters defining the radius vector of the geoid — the zeroth and second degree zonal harmonics. (The more literal definition of the mean equatorial radius as the radius of the circle which best fits an equatorial section through the geoid would connect the radius to the infinite set of even degree zonal harmonics.) An alternative possibility for the equatorial radius in Equation 1 is the mean radius of the entire earth which, since it differs by a factor of  $10^{-3}$ , would affect the value of  $J_2$ . The mean radius seems slightly preferable aesthetically, but current practice overwhelmingly favors the equatorial radius; a perusal of some papers on close satellite dynamics and orbit analysis found ten workers using the equatorial radius but none using the mean radius (in addition, five theoreticians did not define their radius).

To be consistent with the connection of equatorial radius to the mean earth ellipsoid, it is recommended that the following be the relationships between the astronomical parameters  $\mu = GM_\oplus$  and  $J_2 = C_{2,0}$  and the geodetic parameters  $R = a_e$ , the equatorial radius;  $\gamma_e$ , the equatorial gravity;  $f$ , the flattening; and  $\omega$ , the rate of the earth's rotation with respect to inertial space (References 2, 3, and 4):

$$GM_\oplus = a_e^2 \gamma_e \left[ 1 + \frac{3}{2} m - f - \frac{15}{14} mf - \frac{1}{294} mf^2 - O(f^4) \right], \quad (5)$$

$$J_2 = \frac{2}{3} f \left( 1 - \frac{1}{2} f \right) - \frac{1}{3} m \left[ 1 - \frac{3}{2} m - \frac{2}{7} f + \frac{9}{4} m^2 + \frac{11}{49} f^2 + O(f^3) \right], \quad (6)$$

where

$$m = \frac{\omega^2 a_e}{\gamma_e}. \quad (7)$$

The values of  $GM_\oplus$  and  $J_2$  which are probably the most extensively used at orbit computation centers in the United States are (References 5, 6, and 7):

$$\left. \begin{aligned} GM_\oplus &= 3.986032 \pm 0.000030 \times 10^{20} \text{ cm}^3 \text{ sec}^{-2}, \\ J_2 &= 1082.30 \times 10^{-6}. \end{aligned} \right\} \quad (8)$$

In the alternative notation of Herrick, Baker, and Hilton (Reference 8) and Makemson, Baker, and Westrom (Reference 9):

$$k_e = (GM_\oplus)^{1/2} = 0.019965049 \text{ megameter}^{3/2} \text{ sec}^{-1}. \quad (9)$$

The values of  $GM_\oplus$  and  $J_2$  in Equation 8 are consistent with these values for the geodetic parameters:

$$\left. \begin{aligned} a_e &= 6,378,165.0 \pm 25.0 \text{ meters,} \\ \gamma_e &= 978.0300 \pm 0.012 \text{ cm sec}^{-2}, \\ f &= 1/298.30, \\ \omega &= 0.729211585 \times 10^{-4} \text{ sec}^{-1}. \end{aligned} \right\} \quad (10)$$

The value for  $a_e$  is a compromise between the solutions of Fischer (Reference 10), and Kaula (Reference 11), and other values which are unpublished. The  $\gamma_e$  value differs from that of the International Formula and the Potsdam System ( $978.0490 \text{ cm sec}^{-2}$ ) in three ways:

1. Correction to Potsdam System absolute  $g$  (Reference 12) =  $-0.0128 \pm 0.0003$ ;
2. Change of flattening from  $1/297$  to  $1/298.3$  =  $-0.0051$ ;
3. Change of mean gravity over the earth's surface (Reference 11) =  $-0.0005 \pm 0.0012$ .

The correction to absolute  $g$  is a provisional value and has not been adopted by the International Union of Geodesy and Geophysics; an improved value should be forthcoming within the next few years from several determinations in progress (Reference 13). The correction to mean gravity is negative, mainly because correlation between gravity and topography was used to estimate anomalies for the areas without observations, which are predominantly oceans. Solutions by Uotila which fit observed gravimetry and do not use correlation with topography give positive corrections ranging from  $+0.0004$  to  $+0.0019 \text{ cm sec}^{-2}$  (Reference 14). Rather slow improvement is expected; problems in observing gravity at sea are not entirely solved (References 15 and 16). Some improvement may also come from using the better statistical techniques which larger capacity computers permit.

The value of  $GM_\oplus$  may also be obtained through the modified Kepler equation by using the radar mean distance of the moon  $A$  and the moon's mean motion  $n$ :

$$GM_\oplus = \frac{n^2(1+\beta)^3}{1 + \frac{\mu_M}{\mu_E}} A^3, \quad (11)$$

where  $\beta$  is the solar perturbation of the mean semimajor axis and  $\mu_M/\mu_E$  is the ratio of the moon's mass to the earth's mass, equal to the lunar inequality (Reference 17). The most recently published value for  $A$  is  $384,402.0 \pm 1.2 \text{ km}$  (Reference 18). As pointed out by Fischer, this value should perhaps be corrected because it is dependent on an excessively rounded-off lunar radius of  $1740 \text{ km}$  (Reference 19). The mean radius of the lunar limb is  $1737.85 \pm 0.07 \text{ km}$ . Geometrical determinations of the

radius toward the earth vary considerably; Baldwin's conclusion (Reference 20) leads to 1740.05 km, whereas Schrutka-Rechtenstamm (Reference 21) concludes that the bulge is too small to be determined. However, we are not interested in just the long axis of a best-fitting triaxial ellipsoid, but rather in the mean radius of the area contributing to the leading edge of the radar return pulse, which would fall within the  $\pm 7$  degree area of libration. Contour maps of the moon (Reference 22, for example) indicate that the average radius of this  $\pm 7$  degree area could differ by as much as 2 km from the best-fitting ellipsoid. If the lunar surface is assumed to be an equipotential surface, then using the moments of inertia obtained from the physical libration yields 1738.57 km as the radius toward the earth. Letting  $A = 384,400.5 \pm 1.2$  km,  $\beta = 0.0090678$ ,  $n = 2.6616997 \times 10^{-6}$  sec $^{-1}$  (Reference 23), and  $\mu_M/\mu_E = 1/(81.375 \pm 0.026)$  (Reference 24) gives

$$GM_{\oplus} = 3.986094 \pm 0.00004 \times 10^{20} \text{ cm}^3 \text{ sec}^{-2}. \quad (12)$$

Using the  $\mu_M/\mu_E = 1/81.219$  of Delano (Reference 25) reduces  $GM_{\oplus}$  to  $3.986001 \times 10^{20} \text{ cm}^3 \text{ sec}^{-2}$ , so the difference from solutions based on terrestrial data seems largely explicable as an error in the lunar inequality. The larger computers of today permit the application of more elaborate statistical techniques than it was possible to apply in 1950 (the year Delano and Rabe published their work). However, since the stellar positions are a major suspect for systematic error, it seems premature to reanalyze the Eros observations before the revised reference star systems are available (Reference 26). Meanwhile, improved determination of the lunar inequality may be obtained from radio tracking of space probes such as Mariner II (1962  $\alpha\rho 1$ ). Also, since spacecraft have been launched into high, nearly circular orbits such as those of Midas III (1961  $\sigma$ ) and Midas IV (1961  $\alpha\delta$ ), it may be worthwhile to try to determine  $GM_{\oplus}$  from close satellite orbits.

In addition to  $GM_{\oplus}$  and  $J_2$ , standard orbit computation programs usually incorporate  $J_3$  and  $J_4$ . The values which are probably most common at United States computation centers are (Reference 6):

$$\left. \begin{aligned} J_3 &= -2.3 \times 10^{-6}, \\ J_4 &= -1.8 \times 10^{-6}. \end{aligned} \right\} \quad (13)$$

At present the best values of the zonal harmonics are undoubtedly those of Kozai (Reference 7):

$$\left. \begin{aligned} J_2 &= 1082.48 \pm 0.06 \times 10^{-6}, & J_3 &= -2.562 \pm 0.012 \times 10^{-6}, \\ J_4 &= -1.84 \pm 0.08 \times 10^{-6}, & J_5 &= -0.064 \pm 0.019 \times 10^{-6}, \\ J_6 &= 0.39 \pm 0.12 \times 10^{-6}, & J_7 &= -0.470 \pm 0.021 \times 10^{-6}, \\ J_8 &= -0.02 \pm 0.02 \times 10^{-6}, & J_9 &= 0.117 \pm 0.025 \times 10^{-6}. \end{aligned} \right\} \quad (14)$$

Note that the  $J_2$ ,  $J_3$ , and  $J_4$  now used, given in Equations 8 and 13, each differ from Kozai's improved values by less than  $0.3 \times 10^{-6}$ ; and that the coefficients  $J_5$  and higher are all very small in absolute magnitude. Therefore, it does not seem worthwhile to adopt values, other than those already in general use, before 1966 or 1967, when analysis of geodetic satellite orbits observed during the International Year of the Quiet Sun will be completed.

Most of the current close satellite orbit analyses for geodetic purposes seek tesseral harmonic perturbations. In view of the smallness of these perturbations, it does not seem appropriate to adopt standardized values for the tesseral harmonics  $C_{n,m}$ ,  $S_{n,m}$ . The one exception might be  $C_{2,2}$ ,  $S_{2,2}$ , for which an upper limit would be useful because of its effect on supplemental energy requirements for 24 hour orbits. The most recent, unpublished determinations of Izsak, Kaula, Kozai, and Newton range from  $0.9 \times 10^{-6}$  to  $1.8 \times 10^{-6}$  in amplitude  $(\sqrt{C_{2,2}^2 + S_{2,2}^2})$  and from  $8^\circ$  to  $25^\circ$ W in the direction of the principal axis  $\left[ (1/2) \tan^{-1} (S_{2,2}/C_{2,2}) \right]$ .

## GEOMETRICAL PARAMETERS

As shown by analyses involving large systems of observations (References 10, 11, and 19), the equatorial radius is a derived, rather than a fundamental, quantity: accurate knowledge of the radius is not necessary to obtain other parameters, such as the lunar distance, geoid undulations, or datum positions by fitting of the astro-geodetic to the gravimetric geoid. However, for astronomical purposes, it is desirable to have a reference ellipsoid correct within  $\pm 50$  meters in order to obtain reasonably correct positions of isolated tracking stations from astronomic latitude and longitude. Also it is convenient to have a unit of length approximating the earth's radius for use in the potential formula (Equation 1) and for use as a base line to compare or combine parallax observations. For these astronomical purposes, the value of 6,378,165.0 meters given in Equation 10 should be entirely adequate. Marked improvement is not expected for about 5 years, by which time satellite observations should contribute significantly to the strengthening of triangulation systems and to the interconnection of geodetic datums.

By far the most annoying problems in the astronomical application of geodetic data pertain to tracking station positions. Errors in the adopted values of station positions, in conjunction with drag and nonuniform distribution of observations, prevent accurate determination of tesseral harmonics and are even believed to be a major cause of discrepancies in space probe trajectories (Reference 27). These station position errors are due to both inadequate data and mistaken treatment of data; in descending order of reprehensibility they include:

1. Weak, erroneous, or nonexistent connection of tracking stations to local geodetic control (this includes the moving of antennas by stations without informing the computing center);
2. Failure to state the datum or ellipsoid to which tracking station positions refer;
3. Use of obsolete or erroneous standard datum and ellipsoid;
4. An incomplete or ambiguous statement about how datum or ellipsoid transformations were made;
5. Failure to provide for geoid-ellipsoid difference in calculating heights;

6. Neglecting systematic error due to incorrect observation (for example, no Laplace stations) or incorrect adjustment (for example, arbitrary scale changes or rotations) of geodetic control connecting tracking stations more than, say, 1000 km apart;

7. Actual observational error of position.

In view of the number of geodetic datums and corrections thereto, they do not seem to be appropriate parameters to be adopted as standard by an international organization, except possibly for the large continental triangulation systems. The corrections to coordinates  $u, v, w$  with positive axes directed respectively toward latitude and longitude ( $0^\circ, 0^\circ$ ), ( $0^\circ, 90^\circ\text{E}$ ), ( $90^\circ\text{N}$ ) obtained in the world geodetic system solution of Kaula (Reference 11) are listed in Table 1, where NAD, ED, and TD refer to the North American, European, and Tokyo datums, respectively. The uncertainties in this table are based on estimates of the errors due to interpolation and representation in the astro-geodetic and gravimetric geoids, and are probably a fair measure of item 7 on the above list, but may neglect significant effects falling under item 6. The relationships of the rectangular coordinates  $u, v, w$  to the geodetic latitude  $\phi$ , longitude  $\lambda$ , and elevation  $h$ , referred to an ellipsoid of parameters  $a_e$  and  $f$ , are:

$$\left. \begin{aligned} u &= (\nu + h) \cos \phi \cos \lambda, \\ v &= (\nu + h) \cos \phi \sin \lambda, \\ w &= \left[ (1 - e^2) \nu + h \right] \sin \phi, \end{aligned} \right\} \quad (15)$$

where  $\nu = a_e^2 / (1 - e^2 \sin^2 \phi)^{1/2}$  and  $e^2 = 2f - f^2$ .

Table 1  
Corrections to  $u, v, w$  from Reference 11 (meters).

Datum Shift	$\Delta u$	$\Delta v$	$\Delta w$
WGS-NAD	$-23 \pm 26$	$+142 \pm 22$	$+196 \pm 22$
WGS-ED	$-57 \pm 23$	$-37 \pm 29$	$-96 \pm 23$
WGS-TD	$-89 \pm 40$	$+551 \pm 53$	$+710 \pm 40$

To help minimize the number of unnecessary errors in categories 1 through 5 on the above list, it is suggested that organizations be urged to publish the following information pertaining to each tracking station for which they publish any precise observations of artificial satellites or probes, or orbital data based thereon:

1. The names and coordinates of local geodetic control points, both horizontal and vertical, to which the tracking station is connected;
2. The geodetic datum and ellipsoid to which the horizontal coordinates refer;

3. The organization which established the local geodetic control points;
4. The manner in which the horizontal and vertical survey connections were made from the local control points to the tracking station;
5. The date of the survey connection and a description of the termination point of the survey;
6. The geodetic ( $\phi$ ,  $\lambda$ ,  $h$ ) and rectangular ( $u, v, w$ ) coordinates of the station referred to the local geodetic datum;
7. A statement of the geoid height, if any, estimated for the station and the basis for the estimate;
8. If the tracking station position has been shifted for the purpose of referring observations (direction cosines or altitude and azimuth) or calculating orbits, the geodetic and rectangular coordinates after the shift and the ellipsoid to which the new coordinates refer.

Every item on this list is an action which must be accomplished for any tracking station, but thus far the Smithsonian Astrophysical Institute Baker-Nunn camera network is the only one for which even part of the list has been published (Reference 28). It is symptomatic of the difficulties which occur that, since this publication, the coordinates for at least four of the twelve Baker-Nunn cameras have been found to be in error by 20 meters or more. These geometrical details of tracking station position are rather uninteresting, but they must be examined carefully and determined correctly if the full potentialities of modern tracking techniques are to be realized.

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